A non-perturbative mechanism for elementary particle mass generation

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Outline of the talk

1. Motivation & Introduction
   - Unconventional alternative to the Higgs mechanism
2. A Non Perturbative effect
   - Lattice QCD
3. “Naturally” light NP elementary particle masses
   - A toy-model
4. Mass hierarchy
   - The stronger the interaction, the larger the mass
   - Super-strongly interacting particles are predicted
5. Comments, conclusions & outlook
   - Towards a BSM model?

Part I
Motivation & Introduction

Standard Model and beyond ...

1. Higgs discovery: spectacular confirmation of Standard Model
2. Open problems
   - Fermion mass hierarchy ($m_{el} \sim 3 \cdot 10^{-6} m_{top}$, neutrino masses)
   - What sets the electro-weak scale in the $\sim$TeV region?
   - Naturalness 't Hooft

   - Cabibbo-Kobayashi-Maskawa matrix
   - Dark Matter
   - Magnitude of CP violation (matter vs. antimatter)
   - Gravity
3. Beyond-the-Standard-Model models
   - Supersymmetric Standard Model
     - O(100) parameters, too many?
     - How to break Supersymmetry?
     - LHC bounds?
   - Many Standard Model variants
     - *Ad hoc* particle content
     - Don't provide (elegant) explanations to most of the problems above

Standard Model and beyond ...

- SM has O(20) free parameters (too many?)
  - coupling constants
  - quark and lepton masses
  - CKM matrix elements
  - neutrino mass and mixing parameters
- SM describes fermion (& weak boson) masses, but
  - why $m_{quark} > m_{lepton} \gg m_{neutrino}$?
  - why $m_t \sim 80000\, m_u$ ($m_\tau \sim 4000\, m_e$),
    - though t & u (τ & e) have identical quantum numbers?
  - why $v_{SM} \simeq 250$ GeV $\sim m_{Higgs} \sim m_W$ – is this “natural”?
- SM works extremely well
- SM is maybe the LE theory of some more fundamental model
  - SM renormalizability makes very hard to guess what’s beyond
- A deeper understanding of masses, a window on New Physics?

A very unconventional approach ...

- ... where fermion masses are dynamically generated
  - by the similar kind of physics that in QCD makes $\langle \bar{q}q \rangle \neq 0$
  - i.e. a dynamical χSB effect, triggered by some explicit χSB term
  - in LQCD the latter is provided by either mass or Wilson term

1. A "small" fermion mass term visible in (massless) Wilson LQCD
   - underneath the $1/a$ linear divergency
   - separation would require an infinite tuning - "naturalness"
2. A solution to naturalness $\rightarrow$ enlarge QCD to a model where
   - an $\mathrm{SU}(N_f = 2)$ doublet of strongly interacting fermions is coupled to a scalar field via Yukawa & Wilson-like terms
   - at a critical Yukawa coupling, where they (almost) "compensate"
     - the model enjoys a larger symmetry (up to $(\text{cutoff})^{-2}$ effects)
     - that keeps the fermion 'naturally light - 't Hooft
   - the dynamically generated "finite" NP mass is $O(\alpha_s^2 \Lambda_s)$
3. ... but, its precise value depends on UV details
   - non-perturbative violation of universality!

Part II

A Non Perturbative effect

- Lattice QCD

An inspiration from Wilson lattice QCD - I

$$S_F^L = a^4 \sum_x \bar{q}(x)\Big[\frac{\gamma_\mu}{2}(\nabla_\mu + \nabla^*_\mu) - \frac{ar}{2}\nabla_\mu\nabla^*_\mu + m_0\Big]q(x)$$

- Wilson-term breaks χ-sym
- recovered at $m_{cr} = \frac{c_0}{a} + c_1\Lambda_{QCD} + O(a)$

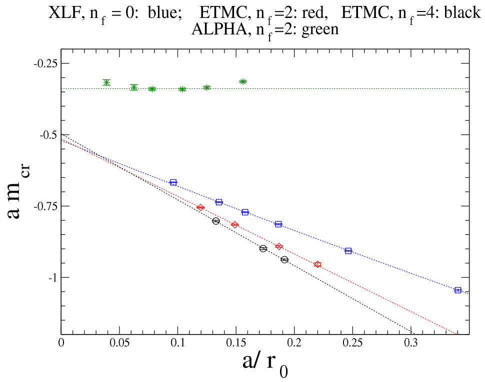
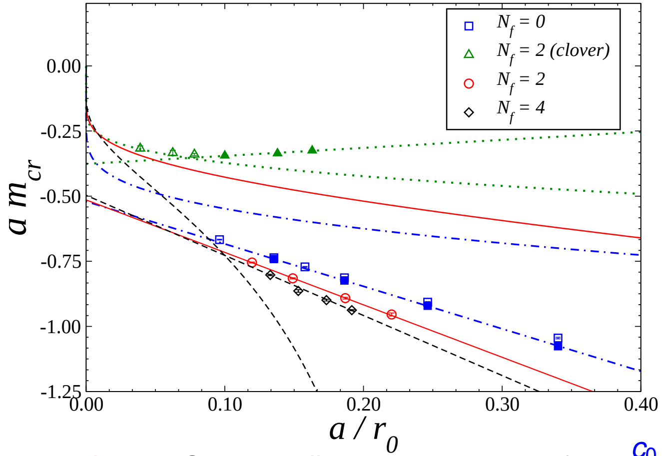


Figure : Green points clover-improved data - Rest are twisted Wilson

An inspiration from Wilson lattice QCD - II

Comparing with two-loop perturbation theory [H. Panagopoulos *et al.*]



Naturalness problem - Can one disentangle $c_1 \Lambda_{QCD}$ from $\frac{c_0}{a}$?

(not really, only one operator $\to \bar{q}q$)

An inspiration from Wilson lattice QCD - III

- If we could, we would get quite an amazing result
- Consider the axial WTI ($f = 1, 2, \ldots, N_f^2 - 1$) [Bochicchio, *et al.* 1985]

$$\nabla_\mu \langle \hat{J}^f_{5\mu}(x) \hat{O}(0) \rangle^L = \langle \Delta^f_5 \hat{O}(0) \rangle \delta(x) + 2(m_0 - \bar{M}(m_0)) \langle P^f(x) \hat{O}(0) \rangle + \mathrm{O}(a)$$

m_{cr} is the value of m_0 at which $m_{PCAC} = m_0 - \bar{M}(m_0) = 0$

- The general expression of $\bar{M}(m_0)$ is something like ($d_1 = \mathrm{O}(g_s^2)$)

$$\bar{M}(m_0) = \frac{c_0(1 - d_1)}{a} + c_1(1 - d_1)\Lambda_{QCD} + m_0 d_1 + \mathrm{O}(a)$$
$$\Longrightarrow m_{cr} = \frac{c_0}{a} + c_1 \Lambda_{QCD} + \mathrm{O}(a)$$

- Setting $m_0 = c_0/a$ (not $m_0 = m_{cr}$) in the WTI, one finds

$$\nabla_\mu \langle \hat{J}^f_{5\mu}(x) \hat{O}(0) \rangle^L = \langle \Delta^f_5 \hat{O}(0) \rangle \delta(x) - 2\underline{c_1(1 - d_1)\Lambda_{QCD}} \langle P^f(x) \hat{O}(0) \rangle + \mathrm{O}(a)$$

$\Longrightarrow$ we seem to be getting a finite (up to logs) fermion mass!

Origin of $c_1 \Lambda_{QCD}$ - Symanzik language

1. in LQCD$_{cr}$ residual O(a) χ-breaking terms induce dynamical χSB
2. the latter in turn brings about O(a) corrections ...
3. ... affecting Lattice quark and gluon propagators and vertices

$$\langle O(x,x',\ldots)\rangle\Big|^{L}_{cr} = \langle O(x,x',\ldots)\rangle\Big|^{C}_{cr} - a\,\langle O(x,x',\ldots)\int d^4z\, L_5(z)\rangle\Big|^{C}_{cr} + \mathrm{O}(a^2)$$

$$O(x,x',\ldots) \Leftrightarrow A^b_\mu(x)A^c_\nu(x')\,,\; q_{L/R}(x)\bar q_{L/R}(x')\,,\; q_{L/R}(x)\bar q_{L/R}(x')A^b_\mu(y)$$

$$\Delta G^{bc}_{\mu\nu} \qquad\qquad \Delta S_{LL/RR} \qquad\qquad \Delta\Gamma^{b,\mu}_{Aq\bar q}$$

- Wilson term $\Longrightarrow$ L_5 $\rightarrow$ χ-violating, d = 5 Symanzik operators
- R_5-even $O \rightarrow a\langle O\int L_5\rangle|^C_{cr} \neq 0$, only because of dynamical χSB
  - would vanish under $R_5 \equiv [q \rightarrow \gamma_5 q\,,\, \bar q \rightarrow -\bar q\gamma_5] \in \mathrm{SU}_R(N_f)\times\mathrm{SU}_L(N_f)$
- dimensional arguments $\rightarrow$ NP $a\,\mathrm{O}(\alpha_s\Lambda_{QCD})$ terms are generated
  - that add up to perturbative propagators and vertices
  - are proportional to the non-analytic $r/|r|$ ratio (typical of SχSB, analog to $m_q/|m_q|$ dependence of $\langle\bar q q\rangle$)

Origin of $c_1 \Lambda_{QCD}$ - Diagrammatic picture

- Gluon propagator

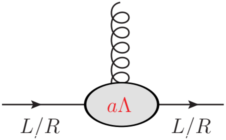
$$\Delta G^{bc}_{\mu\nu}(k)|^L \quad \Longrightarrow$$

$$a \langle A^b_\mu(x) A^c_\nu(x') \int d^4z\, L_5(z) \rangle =$$

$$= a \langle \Omega_0 | A^b_\mu(x) A^c_\nu(x')\, g_s \int d^4z\, (\bar{q}\, \sigma_{\mu\nu} F_{\mu\nu} q)(z)\, g_s \int d^4z'\, (\bar{q} \not{A} q)(z') | \Omega_0 \rangle + \mathrm{O}(g_s^4)$$

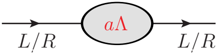
- LL/RR fermion-gluon vertex

$$\Delta\Gamma^{b,\mu}_{Aq\bar{q}}(k,\ell)|^L \quad \Longrightarrow$$



- LL/RR fermion propagator

$$\Delta S_{LL/RR}(k)|^L \quad \Longrightarrow$$



Origin of $c_1 \Lambda_{QCD}$ - O(a) NP terms in lattice correlators

- LL/RR fermion-gluon vertex

$$\Delta\Gamma^{b,\mu}_{Aq\bar{q}}(k,\ell)\Big|^{L} = a\Lambda_{QCD}\alpha_s(\Lambda_{QCD})ig_s\lambda^b\gamma_\mu f_{Aq\bar{q}}\Big(\frac{\Lambda^2_{QCD}}{k^2},\frac{\Lambda^2_{QCD}}{\ell^2},\frac{\Lambda^2_{QCD}}{(k+\ell)^2}\Big)$$

- Gluon propagator

$$\Delta G^{bc}_{\mu\nu}(k)\Big|^{L} = -a\Lambda_{QCD}\,\alpha_s(\Lambda_{QCD})\delta^{bc}\,\frac{\delta_{\mu\nu}-k_\mu k_\nu/k^2}{k^2}\,f_{AA}\Big(\frac{\Lambda^2_{QCD}}{k^2}\Big)$$

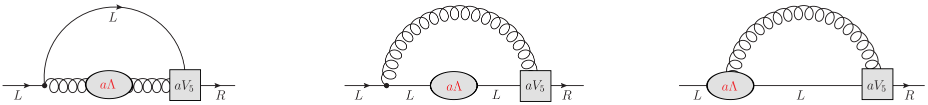
- LL/RR fermion propagator

$$\Delta S_{LL/RR}(k)\Big|^{L} = -a\Lambda_{QCD}\,\alpha_s(\Lambda_{QCD})\frac{ik_\mu(\gamma_\mu)_{LL/RR}}{k^2}f_{q\bar{q}}\Big(\frac{\Lambda^2_{QCD}}{k^2}\Big)$$

$$f_{AA}\Big(\frac{\Lambda^2_{QCD}}{(mom)^2}\Big)\,,\,f_{q\bar{q}}\Big(\frac{\Lambda^2_{QCD}}{(mom)^2}\Big)\,,\,f_{Aq\bar{q}}\Big(\frac{\Lambda^2_{QCD}}{(mom)^2}\Big)\xrightarrow{(mom)^2\to\infty} h_{AA}\,,\,h_{q\bar{q}}\,,\,h_{Aq\bar{q}}$$

Origin of $c_1\Lambda_{QCD}$ - Fermion mass generation

- NP effects bring about new fermion self-energy terms, like



  in diagrams where a Wilson vertex, aV_5, is inserted

- yielding the extra "finite" mass contribution

$$\underline{\underline{m_q^{eff}}} \propto a\Lambda_{QCD} g_s^2 \alpha_s(\Lambda_{QCD}) \int^{1/a} d^4k \frac{k_\mu}{k^2}\frac{1}{k^2} a\, k_\mu \sim \underline{\underline{g_s^2\alpha_s(\Lambda_{QCD})\Lambda_{QCD}}}$$

- quadratic UV divergency exactly compensates explicit a^2 power!
- we expect m_q^{eff} proportional to the non-analytic $r/|r|$ ratio (typical of SχSB - analog to $m_q/|m_q|$ dependence of $\langle\bar{q}q\rangle$)

Part III
“Naturally” light NP elementary particle masses

- A toy-model

Solving the naturalness problem - A toy-model

- Problem
  - disentangle (small/NP) $O(\Lambda)$ from (large/PT) $O(a^{-1})$ effects
- Solution
  - extend QCD to a theory where
    - owing to an enlarged symmetry, "light" fermion masses are "natural"
- Toy-model [$\text{QCD}_{N_f=2}$ + scalar + Wilson] $b^{-1} = \text{UV}$ cutoff

$$\mathcal{L}_{\rm toy}(Q,A,\Phi) = \mathcal{L}_{kin}(Q,A,\Phi) + \mathcal{V}(\Phi) + \mathcal{L}_{Yuk}(Q,\Phi) + \mathcal{L}_{Wil}(Q,A,\Phi)$$

- $\mathcal{L}_{kin}(Q,A,\Phi) = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{Q}_L\gamma_\mu\mathcal{D}_\mu Q_L + \bar{Q}_R\gamma_\mu\mathcal{D}_\mu\, Q_R + \frac{1}{2}\mathrm{Tr}\big[\partial_\mu\Phi^\dagger\partial_\mu\Phi\big]$
- $\mathcal{V}(\Phi) = \frac{\mu_0^2}{2}\mathrm{Tr}\big[\Phi^\dagger\Phi\big] + \frac{\lambda_0}{4}\big(\mathrm{Tr}\big[\Phi^\dagger\Phi\big]\big)^2$
- $\mathcal{L}_{Yuk}(Q,\Phi) = \eta\,\big(\bar{Q}_L\Phi Q_R + \bar{Q}_R\Phi^\dagger Q_L\big)$
- $\mathcal{L}_{Wil}(Q,A,\Phi) = \frac{b^2}{2}\rho\big(\bar{Q}_L\overleftarrow{\mathcal{D}}_\mu\Phi\mathcal{D}_\mu Q_R + \bar{Q}_R\overleftarrow{\mathcal{D}}_\mu\Phi^\dagger\mathcal{D}_\mu Q_L\big)$
  - no power divergent fermion mass term can be generated
  - only $O(b^2)$ corrections
  - toy-model lattice simulations are feasible (and under way)

Symmetries

- $\mathcal{L}_{\rm toy}$ is invariant under the (global) $\chi_L \times \chi_R$ transformations
- $\chi_L : \tilde\chi_L \otimes (\Phi \to \Omega_L \Phi)$, with $\tilde\chi_L : Q_L \to \Omega_L Q_L\,,\ \bar Q_L \to \bar Q_L \Omega_L^\dagger\,,\ \Omega_L \in \mathrm{SU}(2)_L$
- $\chi_R : \tilde\chi_R \otimes (\Phi \to \Phi\Omega_R^\dagger)$, with $\tilde\chi_R : Q_R \to \Omega_R Q_R\,,\ \bar Q_R \to \bar Q_R \Omega_R^\dagger\,,\ \Omega_R \in \mathrm{SU}(2)_R$
- not under "chiral" $\tilde\chi_L \times \tilde\chi_R$ transformations (acting on fermions only)
- key idea $\to$ enforce $\tilde\chi_L \times \tilde\chi_R$ as a(n approximate) symmetry
  - LQCD $\to$ LQCD$_{cr}$
    - $\mathrm{SU}_L \times \mathrm{SU}_R$ "chiral symmetry" is recovered at $m_0 = m_{cr}$ up to $O(a)$
    - d = 5 Wilson and d = 3 mass term "compensating" to that order
    - left-over $O(a)$ chiral SB terms
    - capable of triggering SχSB and generating a NP mass term
  - $\mathcal{L}_{\rm toy} \to \mathcal{L}_{\rm toy}^{cr}$
    - $\tilde\chi_L \times \tilde\chi_R$ "chiral symmetry" is recovered at $\eta = \eta_{cr}$ up to $O(b^2)$
    - d = 6 $\mathcal{L}_{Wil}$ and d = 4 $\mathcal{L}_{Yuk}$ "compensating" to that order
    - left-over $O(b^2)$ chiral ($\tilde\chi$) SB terms
    - capable of triggering D$\tilde\chi$SB and generating a NP mass term

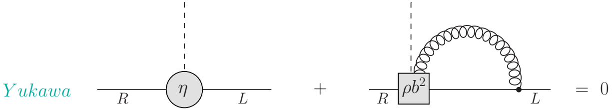
$\tilde\chi_L \times \tilde\chi_R$ transformations and WTIs

- $\tilde\chi_L \times \tilde\chi_R$ WTIs read [Bochicchio *et al.* 1985]
  - $\partial_\mu \langle \tilde J^{L,i}_\mu(x)\,\hat O(0)\rangle = \langle \tilde\Delta^i_L \hat O(0)\rangle \delta(x) - \eta\,\langle O^{L,i}_{Yuk}(x)\,\hat O(0)\rangle - b^2 \langle \underline{O^{L,i}_{Wil}}(x)\,\hat O(0)\rangle$
    - $\tilde J^{L,i}_\mu = \bar Q_L \gamma_\mu \frac{\tau^i}{2} Q_L - \frac{b^2}{2}\rho\Big(\bar Q_L \frac{\tau^i}{2}\Phi \mathcal{D}_\mu Q_R - \bar Q_R \overleftarrow{\mathcal{D}}_\mu \Phi^\dagger \frac{\tau^i}{2} Q_L\Big)$
    - $O^{L,i}_{Yuk} = \Big[\bar Q_L \frac{\tau^i}{2}\Phi Q_R - \mathrm{hc}\Big]$
    - $\underline{O^{L,i}_{Wil}} = \frac{\rho}{2}\Big[\bar Q_L \overleftarrow{\mathcal{D}}_\mu \frac{\tau^i}{2} \Phi \mathcal{D}_\mu Q_R - \mathrm{hc}\Big]$
- Mixing
  - $b^2 \underline{O^{L,i}_{Wil}} = (Z_{\tilde J} - 1)\partial_\mu \tilde J^{L,i}_\mu - \bar\eta(\eta; g_s^2, \rho, \lambda_0)\, O^{L,i}_{Yuk} + \ldots + \mathrm{O}(b^2)$
  - $\partial_\mu \langle Z_{\tilde J} \tilde J^{L,i}_\mu(x)\,\hat O(0)\rangle = \langle \tilde\Delta^i_L \hat O(0)\rangle \delta(x) - (\eta - \bar\eta(\eta))\,\langle O^{L,i}_{Yuk}(x)\,\hat O(0)\rangle + \ldots + \mathrm{O}(b^2)$
- Critical theory $\to \eta - \bar\eta(\eta; g_s^2, \rho, \lambda_0) = 0 \implies \eta_{cr}(g_s^2, \rho, \lambda_0)$
  - $\partial_\mu \langle Z_{\tilde J} \tilde J^{L,i}_\mu(x)\,\hat O(0)\rangle_{\eta_{cr}} = \langle \tilde\Delta^i_L \hat O(0)\rangle_{\eta_{cr}} \delta(x) + \ldots + \mathrm{O}(b^2)$
- All the same with $[L \leftrightarrow R \;\&\; \Phi \leftrightarrow \Phi^\dagger]$
- • Possible NP effects (like slide 10) represented by dots (see below)

- $\mathcal{L}_{\rm toy}^{cr}$ physics dramatically depends on the shape of $\mathcal{V}(\Phi)$
  - If $\mathcal{V}(\Phi)$ has a single minimum $\rightarrow \chi_L \times \chi_R$ realized *á la* Wigner
    - no dynamical $\tilde{\chi}$SB
    - mixings are as we see them in PT
    - NP dots absent in $\tilde{\chi}_L \times \tilde{\chi}_R$ WTIs
    - Φ decouples ($\tilde{\chi}_L \times \tilde{\chi}_R = \chi_L \times \chi_R$ on correlators with $Q's$ & gluons)
    - no generation of NP fermion masses
  - We fix η_{cr} by enforcing $\tilde{J}_\mu^{L,i}$ (or $\tilde{J}_\mu^{R,i}$) conservation

$$x \neq 0 \quad \partial_\mu \langle Z_{\tilde{J}} \tilde{J}_\mu^{L,i}(x)\, O(0)\rangle = (\eta - \bar{\eta}(\eta))\langle [\bar{Q}_L \tau^i \Phi Q_R - \text{hc}](x) O(0)\rangle + \mathrm{O}(b^2)$$

  - which implies in the Wigner phase the cancellation



The critical theory - II

- $\mathcal{L}_{\rm toy}^{cr}$ physics dramatically depends on the shape of $\mathcal{V}(\Phi)$
  - keep the same η_{cr} as determined in the Wigner phase
  - If $\mathcal{V}(\Phi)$ has Mexican hat shape $\to \chi_L \times \chi_R$ realized *á la* NG
    - $\langle\Phi\rangle = v \neq 0$ breaks spontaneously $\chi_L \times \chi_R$ (already at tree-level)
    - $\Phi = v + \sigma + i\vec{\tau}\cdot\vec{\pi}$
    - $\mathcal{L}_{Wil} = \frac{b^2}{2}\rho\big(\bar{Q}_L \overleftarrow{\mathcal{D}}_\mu \Phi \mathcal{D}_\mu Q_R + \mathrm{hc}\big) \stackrel{b^2 v \to ar}{\longrightarrow} \mathcal{L}_{Wil}^{QCD} = -\frac{ar}{2}\big(\bar{Q}_L \mathcal{D}^2 Q_R + \mathrm{hc}\big)$
    - owing to $\eta = \eta_{cr}$, the Yukawa mass term $v\bar{Q}Q$ gets canceled
  - We expect (as in LQCD)
    - dynamical $\tilde{\chi}$SB triggered by residual $\mathrm{O}(b^2 v)$ $\tilde{\chi}$-breaking terms
    - non-vanishing NP dots to affect
      - operator mixing
      - renormalized $\tilde{\chi}_L \times \tilde{\chi}_R$ WTIs
  - NP fermion mass terms (like $c_1\Lambda_{QCD}$ in LQCD) get generated

The critical theory - Wigner vs. NG

- $\partial_\mu \langle \tilde{J}^{L,i}_\mu(x)\, \hat{O}(0)\rangle = \langle \tilde{\Delta}^i_L \hat{O}(0)\rangle \delta(x) - \eta \,\langle \Big[\bar{Q}_L \frac{\tau^i}{2}\Phi Q_R - \mathrm{hc}\Big](x)\, \hat{O}(0)\rangle +$

  $-b^2 \langle \frac{\rho}{2}\Big[\bar{Q}_L \overleftarrow{\mathcal{D}}_\mu \frac{\tau^i}{2}\Phi \mathcal{D}_\mu Q_R - \mathrm{hc}\Big](x)\, \hat{O}(0)\rangle$

- Wigner phase after the tuning $\eta - \bar{\eta}(\eta; g_s^2, \rho, \lambda_0) = 0$
  - $\partial_\mu \langle Z_{\tilde{J}} \tilde{J}^{L,i}_\mu(x)\, \hat{O}(0)\rangle_{\eta_{cr}} = \langle \tilde{\Delta}^i_L \hat{O}(0)\rangle_{\eta_{cr}} \delta(x) + \mathrm{O}(b^2)$

- NG phase $\langle \Phi \rangle = v \to$ "large" Higgs-like mass $v\bar{Q}Q$ drops out
  - $\partial_\mu \langle \tilde{J}^{L,i}_\mu(x)\, \hat{O}(0)\rangle = \langle \tilde{\Delta}^i_L \hat{O}(0)\rangle \delta(x) - \eta_{cr}\, v \,\langle \Big[\bar{Q}_L \frac{\tau^i}{2} Q_R - \mathrm{hc}\Big](x)\, \hat{O}(0)\rangle +$

    $-b^2 v \langle \frac{\rho}{2}\Big[\bar{Q}_L \overleftarrow{\mathcal{D}}_\mu \frac{\tau^i}{2} \mathcal{D}_\mu Q_R - \mathrm{hc}\Big](x)\, \hat{O}(0)\rangle + \Phi\text{-fluctuations} + \ldots + \mathrm{O}(b^2)$

- Is there any mass term left?

Origin of NP effects - Symanzik language

- Consider small-b^2 expansion of a formally $\tilde{\chi}_L \times \tilde{\chi}_R$ inv. correlator

$$\langle O(x,x',...)\rangle\Big|^{R}_{cr} = \langle O(x,x',...)\rangle\Big|^{F}_{cr} - b^2 \langle O(x,x',...) \int d^4 z\, [L_6^{\tilde{\chi}br} + L_6^{\tilde{\chi}co}](z)\rangle\Big|^{F}_{cr} + \mathrm{O}(b^4)$$

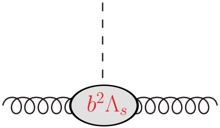
$$O(x,x',...) \Leftrightarrow A_\mu^b A_\nu^c\, \sigma\,,\;\; Q_{L/R}\bar{Q}_{L/R}\,\sigma\,,\;\; Q_{L/R}\bar{Q}_{L/R} A_\mu^b\,\sigma$$

  - $\langle ... \rangle|^R$ = UV-Regulated $\langle ... \rangle|^F$ = Formal correlator
  - $\mathcal{L}_{Yuk} + \mathcal{L}_{Wil} \implies L_6^{\tilde{\chi}br} \to \tilde{\chi}$-violating, d = 6 Symanzik operators
  - $\tilde{R}_5$-even $O \to b^2 \langle O \int L_6^{\tilde{\chi}br} \rangle|^F_{cr} \neq 0$, only due to dynamical $\tilde{\chi}$SB
    - would vanish under $\tilde{R}_5 \equiv [Q \to \gamma_5 Q\,,\, \bar{Q} \to -\bar{Q}\gamma_5] \in \tilde{\chi}_L \times \tilde{\chi}_R$
  - dimensional arguments $\to$ NP $b^2\, \mathrm{O}(\alpha_s \Lambda_s)$ terms get generated
    - that add up to perturbative propagators and vertices
- Key questions
  - how does $\chi_L \times \chi_R$ symmetry constrain dynamical $\tilde{\chi}$SB effects?
  - what plays here the role of the non-analytic $r/|r|$ ratio?

Origin of NP effects - Diagrammatic picture

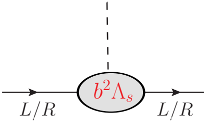
- Gluon-gluon-scalar vertex

$$\Delta\Gamma^{bc\,\mu\nu}_{AA\Phi}\Big|^{R} \quad \Longrightarrow$$



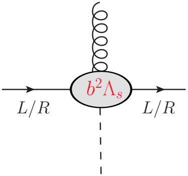
- $Q_{L/R}$-$\bar{Q}_{L/R}$-scalar vertex

$$\Delta\Gamma_{Q\bar{Q}\Phi}\Big|^{R} \quad \Longrightarrow$$



- $Q_{L/R}$-$\bar{Q}_{L/R}$-gluon-scalar vertex

$$\Delta\Gamma^{b\,\mu}_{Q\bar{Q}A\Phi}\Big|^{R} \quad \Longrightarrow$$



Origin of NP effects - NP terms in lattice correlators

Examples of NP corrections: gluon-gluon-scalar, $Q_{L/R}$-$\bar{Q}_{L/R}$-scalar & $Q_{L/R}$-$\bar{Q}_{L/R}$-gluon-scalar vertices ($p = k, \ell, \ell', \dots$)

$$\Delta\Gamma^{bc\mu\nu}_{AA\Phi}(k,\ell)\Big|^{R} = b^2\Lambda_s\alpha_s(\Lambda_s)\frac{\delta^{bc}}{2}\{[k(k+\ell)\delta_{\mu\nu} - k_\mu(k+\ell)_\nu] + [\mu \to \nu]\} F_{AA\Phi}\Big(\frac{\Lambda_s^2}{p^2}\Big)$$

$$\Delta\Gamma_{Q\bar{Q}\Phi}(k,\ell)\Big|^{R} = b^2\Lambda_s\,\alpha_s(\Lambda_s)\,\frac{i}{2}\gamma_\mu(2k+\ell)_\mu\, F_{Q\bar{Q}\Phi}\Big(\frac{\Lambda_s^2}{p^2}\Big)$$

$$\Delta\Gamma^{b,\mu}_{Q\bar{Q}A\Phi}(k,\ell,\ell')\Big|^{R} = b^2\Lambda_s\,\alpha_s(\Lambda_s)\, ig_s\lambda^b\gamma_\mu\, F_{Q\bar{Q}A\Phi}\Big(\frac{\Lambda_s^2}{p^2}\Big)$$

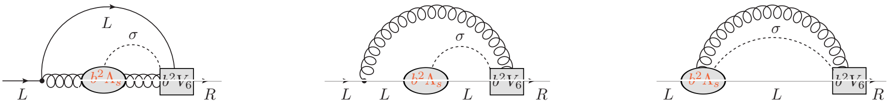
Like in LQCD, we assume NP effects displayed here persist up to $p^2 = \mathrm{O}(b^{-2})$, and *conjecture* the asymptotic behaviour

$$F_{AA\Phi}\Big(\frac{\Lambda_s^2}{p^2}\Big) \stackrel{p^2\to\infty}{\longrightarrow} H_{AA}\,,\quad F_{Q\bar{Q}\Phi}\Big(\frac{\Lambda_s^2}{p^2}\Big) \stackrel{p^2\to\infty}{\longrightarrow} H_{Q\bar{Q}}\,,\quad F_{Q\bar{Q}A\Phi}\Big(\frac{\Lambda_s^2}{p^2}\Big) \stackrel{p^2\to\infty}{\longrightarrow} H_{Q\bar{Q}}$$

with H_{AA} & $H_{Q\bar{Q}} \to \mathrm{O}(1)$ constants

Origin of NP effects - Fermion mass generation

- self-energy diagrams like



- give (e.g. central panel)

$$\underline{\underline{m_Q^{eff}}} \propto g_s^2\alpha_s(\Lambda_s)\int^{1/b}\frac{d^4k}{k^2}\frac{\gamma_\mu k_\mu}{k^2}\int^{1/b}\frac{d^4\ell}{\ell^2+m_\sigma^2}\frac{\gamma_\nu(k+\ell)_\nu}{k^2+\ell^2}\cdot$$
$$\cdot b^2\gamma_\rho(k+\ell)_\rho\, b^2\Lambda_s\gamma_\lambda(2k+\ell)_\lambda \sim \underline{\underline{g_s^2\alpha_s(\Lambda_s)\Lambda_s}}$$

[b^4 factor compensated by the two-loop quartic divergency]

- a NP mass term arises $\Longrightarrow C_1\Lambda_s[\bar{Q}_L Q_R + \bar{Q}_R Q_L]$
- to leading order in $g_s^2 \Longrightarrow C_1\Big|_{LO} = k_{LO}\, g_s^4$

Origin of NP effects - Correlators and WTIs

A few observations

- There are NP effects also in other correlation functions
- We can interpret the above NP terms as *bona fide* masses if
  1. "large" $O(v \gg \Lambda_s)$ mass terms are absent
  2. NP masses can be embodied in a $\chi_L \times \chi_R$ invariant term
  3. the chiral variation of which can be accommodated in the $\tilde{\chi}$-WTIs
- Indeed
  1. true in the critical theory, owing to $\eta = \eta_{cr}$ (see fig. in slide 26)
  2. one can construct χ-inv. "mass term" $C_1\Lambda_s[\bar{Q}_L U Q_R + \bar{Q}_R U^\dagger Q_L]$
     - a term like $m[\bar{Q}_L Q_R + \bar{Q}_R Q_L]$ is not $\chi_L \times \chi_R$ invariant
  3. the renormalized WTIs that embody NP mass terms read

$$\bullet\ \partial_\mu \langle Z_{\tilde{J}} \tilde{J}^{L\,i}_\mu(x)\, \hat{O}(0)\rangle\Big|_{\eta_{cr}} = \langle \tilde{\Delta}^i_L \hat{O}(0)\rangle\Big|_{\eta_{cr}} \delta(x) \ +$$

$$+C_1\,\Lambda_s \,\langle\, [\bar{Q}_L \frac{\tau^i}{2} U Q_R - \mathrm{hc}](x)\, \hat{O}(0)\,\rangle\Big|_{\eta_{cr}} + O(b^2)$$

... where we have introduced U

U field is a non-analytic function of Φ

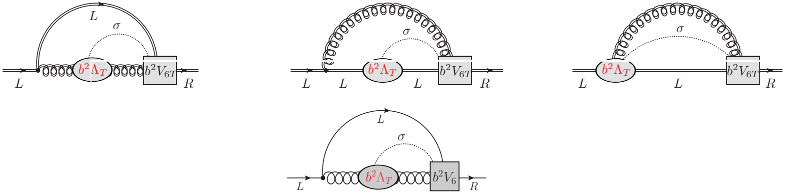
$$U = \frac{\Phi}{\sqrt{\Phi\Phi^\dagger}}$$

- U is the phase of Φ
- U transforms like Φ under $\chi_L \times \chi_R$
- $[\bar{Q}_L U Q_R + \bar{Q}_R U^\dagger Q_L]$ invariant under $\chi_L \times \chi_R$ transformations
- such a “generalised” mass term only possible thanks to U
- U only exists if $\langle\Phi\rangle \neq 0 \Rightarrow$ no NP mass/mixings in Wigner phase

- Wilson r is elevated to a field
- $r/|r| \to U$ a signature of NP effects

Part IV
Mass Hierarchy

- Q_T feel super-strong & strong interactions, q only strong ones
- $q \to N_g = 3$ generations - gauge group SU($N_c = 3$)
- $Q_T \to 1$ generation - gauge groups SU($N_c = 3$) $\times$ SU($N_T = 3$)
- $\beta^0_T/\beta^0_{QCD} = \frac{11N_T - 4N_c}{11N_c - 4N_g - 4N_T} = \frac{7}{3} \quad \Rightarrow \quad \Lambda_T \gg \Lambda_{QCD}$



- leading order vs. RG-improved estimates ($g^2_{s,T} \to g^2_{s,T}(\Lambda_T)$)

$$m_{Q_T}(\Lambda_T) \sim Z_{Q_T} g^2_T(\Lambda_T)\,\alpha_T(\Lambda_T)\,\Lambda_T \qquad m_q(\Lambda_T) \sim Z_q g^2_s(\Lambda_T)\,\alpha_s(\Lambda_T)\,\Lambda_T$$

$$\frac{m_q}{m_{Q_T}}\Big|_{\Lambda_T} \sim \frac{\alpha^2_s(\Lambda_T)}{\alpha^2_T(\Lambda_T)} \sim \frac{1}{10} \div \frac{1}{100} \;\&\; q = \text{top} \Rightarrow m_{Q_T} \sim \text{few TeV's} \sim \underline{\underline{\Lambda_T \gg \Lambda_{QCD}}}$$

Part V
Comments, conclusions & outlook

Comments

1. NG bosons (π^1, π^2, π^3) associated to classical S$\chi_L \times \chi_R$SB
   - can electro-weak interactions be accommodated?
     - gauging of χ_L, with π^1, π^2, π^3 eaten up by weak-bosons
     - $M_{W/Z} \propto g_W \Lambda_T$
2. "NG bosons" with $m^2 \propto \rho^2 \leftrightarrow$ dynamical $\tilde{\chi}$SB (similarly to QCD)
   - Super-strong bound states with $M \sim \Lambda_T$
3. NP masses do not depend on the value of $v = \langle \Phi \rangle$
   - a natural choice is to take $v \gg \Lambda_T$ (otherwise, why not a Higgs?)
   - possibly then $v \sim O(\Lambda_{GUT})$ (recall $v \sim m_\sigma \gg \Lambda_T$)
4. NP masses depend on details of $\tilde{\chi}$-breaking terms at UV scale
   - universality violation at NP level $\Longrightarrow$ check by simulations!
   - is $b^{-1} \gg v$ a finite, physical UV cutoff (maybe $b^{-1} \sim \Lambda_{Planck}$)?
5. The presence of super-strong dof's significantly improves unification of couplings

Conclusions

- We have identified a NP mechanism for mass generation that
  - allows an understanding of fermion mass hierarchy
  - points at a super-strong interaction at a few TeV's scale
- The conjectured scheme is falsifiable by lattice simulations
  - Toy-model with $\Lambda_s \ll v \ll b^{-1}$
  - Strongly + Super-strongly interacting fermions
- The mechanism gives the "naturalness" problem a NP solution
  - exportable to realistic extensions of the SM?
  - need a good&convincing interpretation of 125 GeV resonance
    - maybe is a WW/ZZ-bound state with $E_b = \mathrm{O(tenths)~GeV}$
  - need to study how the "low energy theory" deviates from SM

Outlook

1. A road to a full Beyond-the-Standard-Model model?
   - add to QCD super-strongly interacting Quarks and Leptons
     - no need for Extended Techni-Color
   - introduce electro-weak interactions by gauging χ_L
   - introduce families and split quark & lepton weak isospin doublets
   - all masses $\propto$ ($\Lambda_T \times$ powers of coupling constants)
   - at this stage neutrinos are massless
2. Phenomenology
   - problems?
     - FCNCs to a comfortably low level
     - no fast proton decay
   - comparing with SM (issues in order of decreasing energy)
     - strong and electro-weak coupling unification
     - techni-hadrons with $M_{H_T} \sim$ a few TeV's
     - "low energy theory" looks similar to the SM with $y_y \propto m_f$, but in a non-linear realisation of $SU(2)_L \times U_Y(1)$

Thank you for your attention