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EMFT and Applications

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Lattice Gauge Theory for the LHC and Beyond
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Outline

1) Complex ϕ^4 with a chemical potential
   - The nonzero chemical potential μ introduces a sign problem which prohibits Monte Carlo simulations in the standard representation
   - In EMFT the sign problem can be rotated away and results can be obtained for arbitrary μ and temperature T.
2) Higgs-Yukawa model with higher dimension operators
   - The Higgs-Yukawa model contains the Higgs field and the fermions of the Standard Model.
   - The higher dimension operator is a "generic" BSM induced feature.
   - We show that EMFT can, in some aspects, beat simulations with dynamical chiral fermions and massless Goldstone bosons.
   - We investigate the $T = 0$ and finite-temperature phase diagrams, relevant for electroweak baryogenesis.

1) Complex ϕ^4 with a chemical potential

arXiv:1405.6613

We want to study a relativistic Bose gas in $3+1$ dimensions with the Euclidean Lagrangian density

$$\mathcal{L}[\varphi(x)] = \partial_\nu \varphi^*(x)\partial_\nu \varphi(x) + \left(m_0^2 - \mu^2\right)|\varphi(x)|^2 + \lambda\,|\varphi(x)|^4 + \mu j_0(x)$$

The chemical potential μ couples to the temporal component of the conserved current

$$j_\nu(x) = \varphi^*(x)\partial_\nu\varphi(x) - \partial_\nu\varphi^*(x)\varphi(x),$$

i.e. the charge

$$Q = \int \mathrm{d}^3\vec{x}\; j_0(\vec{x}).$$

$$\mathcal{L}[\varphi(x)] = \partial_\nu \varphi^*(x) \partial_\nu \varphi(x) + \left(m_0^2 - \mu^2\right) |\varphi(x)|^2 + \lambda |\varphi(x)|^4 + \mu j_0(x)$$

- This model has a global $U(1)$ symmetry and exhibits spontaneous symmetry breaking to a Bose-condensed phase at zero temperature when the chemical potential reaches the renormalized mass m_R.
- Due to the complex action the model suffers from a sign problem, which can however be solved by a clever change to “dual” variables [1].
- Like the real ϕ^4 model it is also amenable to EMFT (and DMFT) treatment, which allows a mapping of the full (T, μ)-phase diagram at a very low computational cost.

[1] C. Gattringer and T. Kloiber, Nucl. Phys. B 869 (2013) 56

The EMFT effective action

Analogous to the real case, the effective action contains an external field $\phi \in \mathbb{R}$ and shifts of the quadratic term Δ_1 and Δ_2 (remember, $K^{-1}_{\text{imp,c}}$ contributes only a contact term).

$$S_{\text{EMFT}} = (\eta - \Delta_1)\,\varphi_1^2 + (\eta - \Delta_2)\,\varphi_2^2 + \lambda\left(\varphi_1^2 + \varphi_2^2\right)^2 - 2\phi\varphi_1(2(d - 1 + \cosh(\mu)) - \Delta_1),$$

with $\eta = m_0^2 + 8$ and $\varphi = \varphi_1 + i\varphi_2$. We have used the global $U(1)$ symmetry to align the expectation value to the real axis.

Self-consistency equations

$$S_{\text{EMFT}} = (\eta - \Delta_1)\,\varphi_1^2 + (\eta - \Delta_2)\,\varphi_2^2 + \lambda\left(\varphi_1^2 + \varphi_2^2\right)^2 - 2\phi\varphi_1(2(d - 1 + \cosh(\mu)) - \Delta_1)$$

There are three coupled self-consistency equations in the first and second moments that need to be solved iteratively

$$\langle\varphi\rangle = \phi,$$

$$2\left\langle\varphi_1^2\right\rangle_c = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{\frac{1}{2\left\langle\varphi_1^2\right\rangle_c} + \Delta_1 - 2\kappa Z_h \sum_\nu \cos\left(k_\nu - i\mu\delta_{\nu,t}\right)},$$

$$2\left\langle\varphi_2^2\right\rangle = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{\frac{1}{2\left\langle\varphi_2^2\right\rangle} + \Delta_2 - 2\kappa Z_h \sum_\nu \cos\left(k_\nu - i\mu\delta_{\nu,t}\right)}.$$

Wave-function renormalization

- When going from DMFT to EMFT, the nonperturbative mass renormalization is kept but the wave-function renormalization is lost.
- This can be remedied in the broken phase if the theory contains Goldstone bosons.
- We add an additional parameter Z_h to the self-energy self-consistency equation and fix it such that the Goldstone bosons are exactly massless for an infinite spatial volume.
- Instead of just taking the self-energy Σ_{EMFT} to be constant we make the following substitution

$$\widetilde{\Sigma}(k) \to \Sigma_{\mathrm{EMFT}} + (Z_h - 1) \sum_{\nu=1}^{d} \cos\left(k_\nu - i\mu\delta_{\nu,t}\right)$$

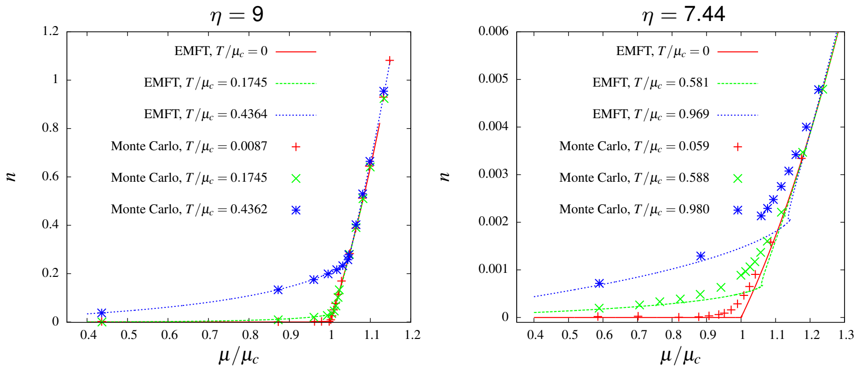
Scale setting and finite temperature

- The scale is set by matching some observable to an experimental value, for example an expectation value (in the broken phase only) or a mass.
- Finite temperature is then trivially introduced by limiting the number of lattice sites in the temporal direction, i.e. by the substitution

$$\int_{-\pi}^{\pi} \frac{\mathrm{d}k_t}{2\pi} \rightarrow \frac{1}{N_t} \sum_{n=0}^{N_t-1}.$$

The charge density

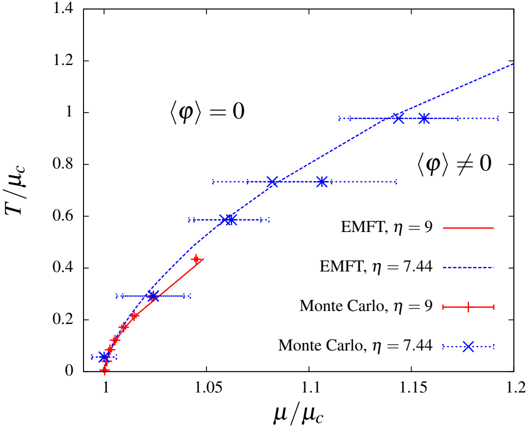
We measure the charge density $n = \partial \log Z / \partial \mu$ and compare to Monte Carlo results [1] (obtained in sign-problem free formulation)



[1] C. Gattringer and T. Kloiber, Nucl. Phys. B 869 (2013) 56

The (T, μ)-phase diagram

The full phase diagram can be obtained within less than an hour and lies well within the error bars of the Monte Carlo result.



2) The Higgs-Yukawa model

- Poor man's version of the Standard Model which nonetheless captures the nonperturbative chiral Higgs-top interaction.
- The Higgs and Yukawa parts of the Lagrangian are given by:

$$\mathcal{L}_{\rm H} = |\partial_\mu \phi|^2 + m_0^2\,|\phi|^2 + \lambda_4\,|\phi|^4 + M_{\rm BSM}^{-2}\,|\phi|^6$$

$$\mathcal{L}_{\rm tb} = \overline{\Psi}_t \partial\!\!\!/ \Psi_t + y_b \overline{\Psi}_{t,{\rm L}} \phi b_{\rm R} + y_t \overline{\Psi}_{t,{\rm L}} \widetilde{\phi} t_{\rm R} + {\rm h.c.} \qquad \widetilde{\phi} = i\tau_2 \phi^\dagger$$

where $\Psi_t = (t, b)^\intercal = (t_{\rm L}, t_{\rm R}, b_{\rm L}, b_{\rm R})^\intercal$ and $\Psi_{t,{\rm L}} = (t_{\rm L}, b_{\rm L})^\intercal$.
To ensure chiral fermions we use the overlap operator.

Benchmarking the EMFT approximation

- To check that the fermions are treated correctly we compare to full Monte Carlo simulations of the same Higgs-Yukawa model [1].
- Due to the large scale separation $M_{\mathrm{BSM}} \gg \langle \varphi \rangle = 246$ GeV and the Goldstone bosons, Monte Carlo simulations suffer from prohibitive finite size effects. With EMFT, infinite volume is available at no extra cost.
- Moreover, the Monte Carlo simulation suffers from a sign problem unless the fermions are mass degenerate ($m_t = m_b$) whereas EMFT can handle the physical case.

[1] P. Hegde, K. Jansen, C. -J. D. Lin and A. Nagy *PoS LATT13* [arXiv:1310.6260]

$\lambda_6 \equiv (aM_{\mathrm{BSM}})^{-2} = 0.1$, “perturbative”

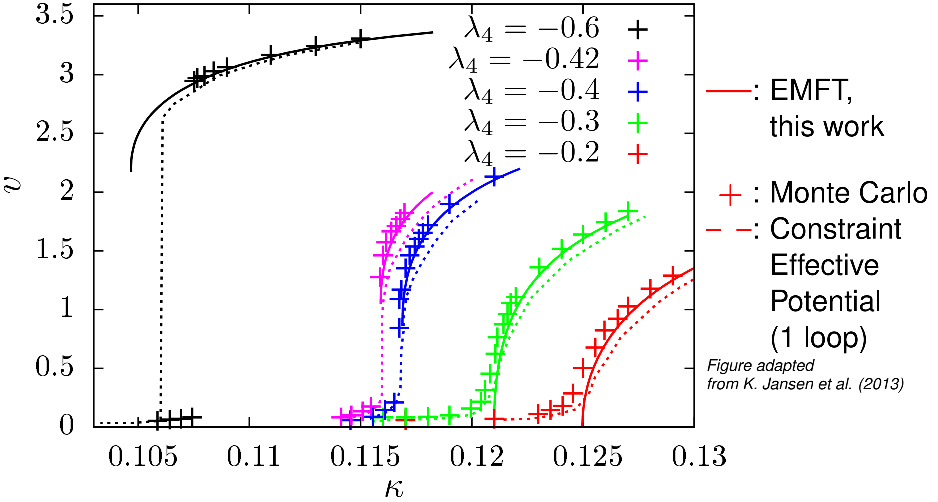


Figure adapted from K. Jansen et al. (2013)

$\lambda_6 \equiv (aM_{\text{BSM}})^{-2} = 1$, "non-perturbative"

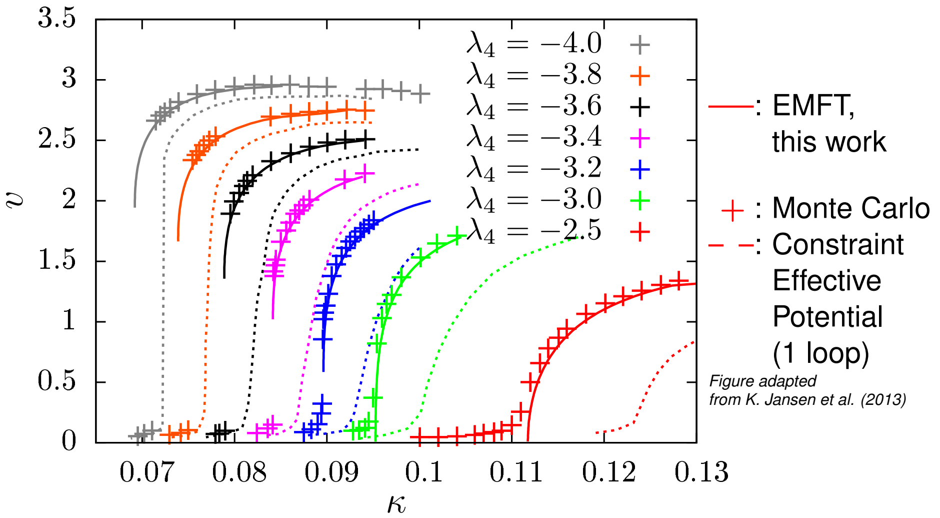
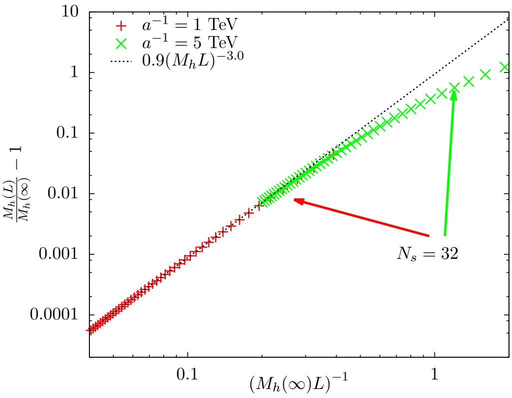


Figure adapted from K. Jansen et al. (2013)

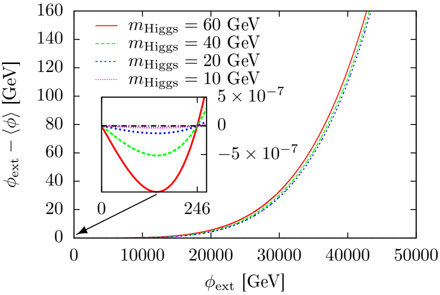
Finite volume effects

The finite volume corrections to the Higgs mass goes like $(M_h L)^{-3}$, which calls for large lattices.



Lowering the Higgs mass bound

$\sim$Derivative of the effective potential, $M_{\mathrm{BSM}} = 50$ TeV

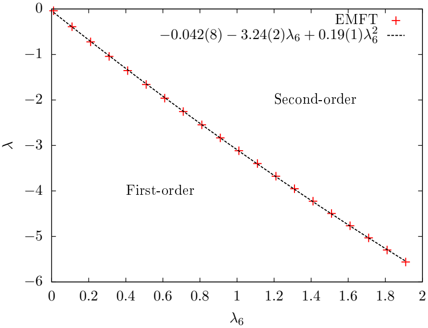


- 2 solutions to $\langle\phi\rangle = \phi_{\mathrm{ext}}$ ($\langle\phi\rangle = 0$ unstable)
- $m_{\mathrm{Higgs}} \approx$ slope at fixed point, can be reduced to $\lesssim 10$ GeV.
- No spurious solution at $\langle\phi\rangle \sim M_{\mathrm{BSM}}$.

Even when M_{BSM} is as heavy as 50 TeV, m_{Higgs} can be as small as 10 GeV

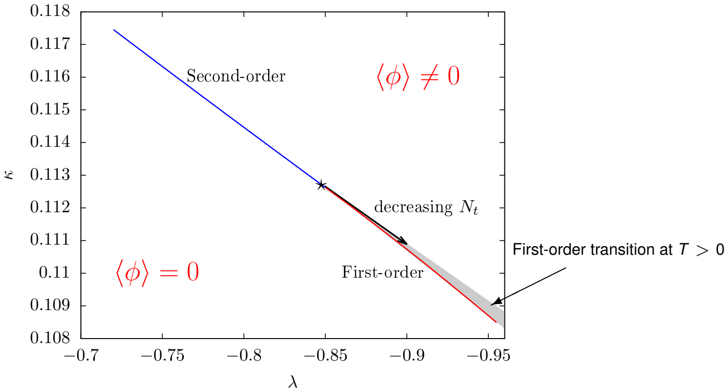
Zero temperature phase diagram

For each $\lambda_6 \equiv (aM_{\text{BSM}})^{-2}$ the κ-driven transition turns first order at a tri-critical quartic coupling λ.



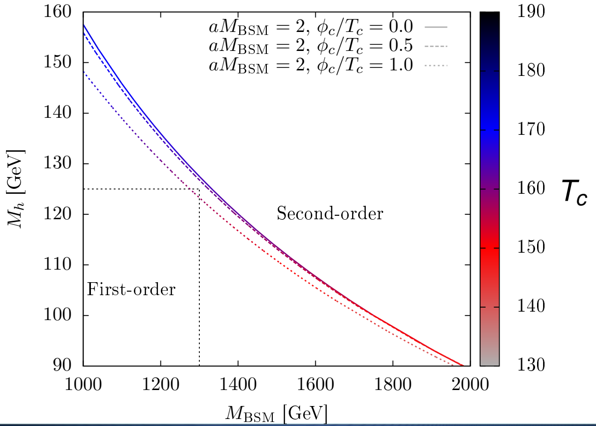
Tri-critical point at finite temperature

For a fixed λ_6 the phase boundary in the (λ, κ)-plane marks the region where one can find a theory with small lattice spacing and also where to expect a first order finite temperature transition.



Finite temperature phase diagram

For fixed Higgs mass M_h the transition turns from second to first order as M_{BSM} decreases.



Conclusions and outlook

- EMFT works provides excellent accuracy at a negligible computational cost.
- It is applicable to models with sign problem and chiral fermions.
- It may be possible to have a strong first order electroweak finite temperature transition if new physics come in at $1-2$ TeV.
  $\rightarrow$ Electroweak baryogenesis.
- The obvious improvement to EMFT (DMFT) is to use a cluster of live sites $\Rightarrow$ Gauge fields?

Thank you for your attention!